

# On the Nuclear Modification Factor at RHIC and LHC

Andrey Kormilitzin,<sup>1</sup> Eugene Levin,<sup>1,2</sup> and Amir H. Rezaeian<sup>2</sup>

<sup>1</sup>*Department of Particle Physics, Tel Aviv University, Tel Aviv 69978, Israel*

<sup>2</sup>*Departamento de Física, Universidad Técnica Federico Santa María,  
Avda. España 1680, Casilla 110-V, Valparaíso, Chile*

(Dated: May 18, 2011)

We show that pQCD factorization incorporated with pre-hadronization energy-loss effect naturally leads to flatness of the nuclear modification factor  $R_{AA}$  for produced hadrons at high transverse momentum  $p_T$ . We consider two possible scenarios for the pre-hadronization: In scenario 1, the produced gluon propagates through dense QCD medium and loses energy. In scenario 2, all gluons first decay to quark-antiquark pairs and then each pair loses energy as propagating through the medium. We show that the estimates of the energy-loss in these two different models lead to very close values and is able to explain the suppression of high- $p_T$  hadrons in nucleus-nucleus collisions at RHIC. We show that the onset of the flatness of  $R_{AA}$  for the produced hadron in central collisions at midrapidity is about  $p_T \approx 15$  and 25 GeV at RHIC and the LHC energies, respectively. We show that the smallness ( $R_{AA} < 0.5$ ) and the high- $p_T$  flatness of  $R_{AA}$  obtained from the  $k_T$  factorization supplemented with the Balitsky-Kovchegov (BK) equation is rather generic and it does not strongly depend on the details of the BK solutions. We show that energy-loss effect reduces the nuclear modification factor obtained from the  $k_T$  factorization about  $30 \div 50\%$  at moderate  $p_T$ .

## I. INTRODUCTION

The nuclear modification factor (NMF)  $R_{AA}$  is defined as

$$R_{AA} \equiv \frac{1}{N_{coll}} \frac{\frac{d^2 N_{AA}}{dy d^2 p_\perp}}{\frac{d^2 N_{NN}}{dy d^2 p_\perp}}, \quad (1)$$

where  $N_{coll}$  denotes the number of binary collisions in nucleus-nucleus (AA) collisions. The measurements in  $Au + Au$  collisions at RHIC showed that the value of  $R_{AA}$  is small and constant up to high transverse momentum of produced hadrons in central collisions [1–5], see Fig. 1. At first sight such a behavior at high- $p_T$  contradicts the perturbative QCD (pQCD) factorization theorem [7–11]. Accordingly to the factorization theorem, the inclusive gluonic-jet production cross-section with a large transverse momentum  $p_T$  is proportional to  $A^2 \sigma_{\text{hard}} x G_p^2(x, p_T)$  which leads to  $R_{AA} \rightarrow 1$  (see Sec. III for the details). On the other hand in the Color Glass Condensate (CGC) approach due to gluon saturation and the appearance of a new dimensional scale, saturation momentum  $Q_s$  [12–16], there is a priori no reason to expect the pQCD factorization theorem to be valid. Indeed, it has been proven [17–22] that the factorization theorem can be replaced by the  $k_T$  factorization [23–26] for scatterings of dilute-dilute or dilute-dense system of partons (like deep inelastic scattering with nuclei or scattering of two virtual -photons). For the case of scatterings of dense-dense systems when we have three scales: two saturation momenta for two nuclei and the transverse momentum of produced jet, the  $k_T$  factorization has not been proven yet. However, for understanding of the NMF behavior, it is enough to discuss the behavior of the inclusive hadron production at transverse momentum  $p_T$  larger than both saturation scales. In this kinematic region, the  $k_T$  factorization works [17]. Having this in mind we can assume that the  $k_T$  factorization works in all kinematic regions for scattering of dense-dense systems.

Based on the  $k_T$  factorization, the inclusive production for gluon (jet) in AA collisions at midrapidity can be calculated from the following equation,

$$\frac{d\sigma}{dy d^2 p_T} \Big|_{y=0} = \frac{2C_F}{\alpha_s 2(2\pi)^3} \frac{1}{x_\perp^2} \int d^2 b d^2 B \int_{-\infty}^{+\infty} dz e^{-z} J_0 \left( e^{\frac{1}{2}z} x_\perp \right) \nabla_z^2 N_G(z; b) \nabla_z^2 N_G(z; |\vec{b} - \vec{B}|), \quad (2)$$

where  $p_T$  and  $y$  are the transverse momentum and rapidity of the produced gluon, with notations  $z = \ln(r^2 Q_s^2)$ ,  $x_\perp = p_T/Q_s$  and  $N_G = 2N - N^2$ . We defined  $C_F = (N_c^2 - 1)/2N_c$  where  $N_c$  denotes the number of colors and  $\alpha_s$  is the strong coupling. The forward dipole-nucleus amplitude  $N$  can be obtained via solving the Balitsky-Kovchegov equation (BK)[15]. It is seen that Eq. (2) has  $x_\perp$ -scaling behavior in the kinematic region that the forward dipole amplitude  $N$  has the geometric-scaling property.

Using Eq. (2) we can re-write the NMF in the following form:

$$\text{NMF} \equiv \frac{1}{A^2} \frac{S_A^2}{S_p^2} \frac{\mathcal{T}(x_\perp)}{\mathcal{T}\left(x_\perp \frac{Q_{s,A}}{Q_{s,N}}\right)}, \quad (3)$$

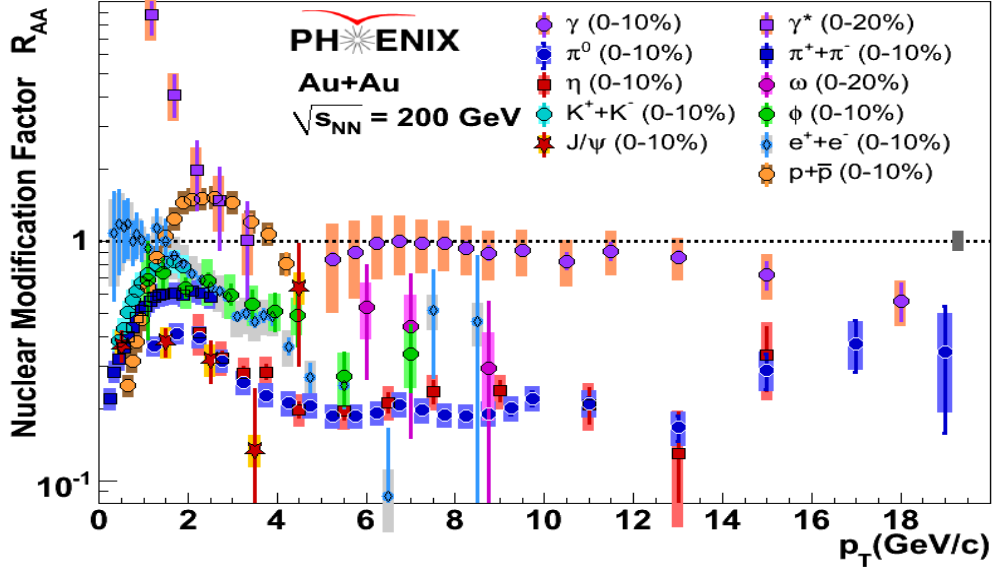


FIG. 1: The NMF for hadrons and direct photon versus transverse momentum  $p_T$  in central Au+Au collisions at RHIC. The plot is taken from Ref. [6] from the PHENIX collaboration.

where function  $\mathcal{T}$  is defined using Eq. (2). The area of interaction (denoted by  $S_A$  and  $S_p$  for AA and  $pp$  collisions) is well-defined for nucleus-nucleus scatterings while is rather uncertain for proton-proton interactions [27, 28]. In the above for simplicity we assumed that the area of interaction in both AA and  $pp$  collisions can be factorized.

It turns out that the NMF calculated from Eq. (3) is small  $R_{AA} < 0.5$  (see Sec IV). Eq. (2) has the geometric scaling behavior which is valid in the saturation region [29] and can be extended to a wider region outside the saturation domain [30]. The geometric scaling behavior is violated for  $z^2 > 2 \ln(Q_s^2(Y)/Q_s(0))$  where  $Y = \ln(1/x)$  [30]. Notice that we have  $2 \ln(Q_s^2(Y)/Q_s(0))$  instead of  $8 \ln(Q_s^2(Y)/Q_s(0))$  since  $d^2\sigma/d^2p_t \propto N_G^2 \propto N^4$  when  $N$  is not small. Therefore, for  $z \gg \sqrt{2 \ln(Q_s^2(Y)/Q_s(0))}$  we expect that the scaling behavior will be broken and the inclusive cross section for jet production for both nucleus-nucleus and proton-proton collisions will be proportional to  $\alpha_s(p_T^2)/p_T^4$  and consequently we have  $R_{AA} \rightarrow 1$ . Using the above equation and the saturation scale estimated in Ref. [31] at RHIC energies, one may expect that only for  $p_T \leq 3 \div 4 Q_s$ , the NMF to be different from unity. But a slight glance at Fig. 1 shows that this is not the case for inclusive hadrons production for a wide range of  $p_T$  while it apparently holds for the production of direct photon [32–34]. Therefore, the small value of the NMF at high  $p_T$  may stem from large distance interactions in medium and non-perturbative QCD should be invoked to calculate such interactions [35]. The data on  $R_{AA}$  for  $J/\Psi$  production supports this conclusion [37–40].

In this paper, we show that the CGC predicts considerable suppression of the NMF at low- $p_T$  both at RHIC and the LHC energies. We will show that based on the CGC prescription at low- $p_T$ , the NMF slowly increases and then flattens at moderate  $p_T$ . Based on pQCD, we will then estimate, the onset of the flatness of the NMF at large  $p_T$ . The CGC approach is based on classical gluon fields and evolution in leading  $\log(1/x)$  approximation. In such approximations the energy-loss in processes of gluon emissions is neglected. The systematic approach within pQCD framework [35, 41–46] shows that such emissions alone is not able to describe the experimental values for the NMF and variety of non-perturbative approaches have been developed [46–54]. In this paper, we introduce a simple approach for calculating the energy-loss effect due to pre-hadronization which is able to describe RHIC data at high- $p_T$ . At low- $p_T$ , energy-loss is less important and we show that the CGC prescription Eq. (3) gives rather good description of RHIC data at small Bjorken- $x$ .

Our approach is based on two main ingredients: (i) the  $k_T$  factorization [17–22] which takes into account the rescatterings of the produced dipole (or gluon) in the final state; and (ii) the energy loss due to the hadronization of the produced dipole (or gluon).

In the next section, we calculate the energy-loss effect within two different pre-hadronization pictures. In Sec. III, we discuss the large  $p_T$  behavior of the NMF for hadron production at RHIC and the LHC. Sec. IV is devoted to present our numerical results and comparison with the experimental data. As a conclusion, in Sec. IV we highlight our main results.

## II. PRE-HADRONIZATION MODELS

Hadronization, unfortunately, could be treated mostly phenomenologically due to our lack of understanding of non-perturbative QCD. There have been several approaches to describe hadronization processes [46–54].

Here, we propose a simple picture for hadronization hoping that the phenomenological uncertainties can be reduced to few parameters which can be then determined from experiment. In our approach, a high energy heavy ion collision can be simplified to three successive stages: 1) parton production 2) pre-hadronization and 3) fragmentation of the produced parton into hadron. At the first stage, we have partons scattering and recombination effects which can be described within the CGC approach. The CGC gives the description of the initial wavefunction of projectiles and the multiple-parton production within a time scale of the order of  $1/Q_s$  after collisions. In the second stage, the produced dipoles propagate, emit gluons and interact with the produced QCD medium and lose energy. As we have already mentioned, the emission of gluons and interactions with other dipoles have been taken into account within the  $k_T$  factorization if the sizes of dipoles are small enough to trust the perturbative QCD approach. In the CGC approach most of produced dipoles have sizes of the order of  $1/Q_s \ll 1/\mu$  where  $\mu$  is the soft scale. The energy loss for such dipoles can be taken into account by the BK equation with a kernel that includes the energy conservation. The estimates of Ref. [65] shows that these corrections cannot explain the experimental data leading only to 20–30% suppression of the NMF. We believe that the main source of the energy loss at this stage of process is originated from hadronization of the slow dipoles that can be created during interaction of the fast dipole with the slow remnants of the nuclear target or with the fast remnants of the projectile nucleus. In the lab. frame one of the nucleus is at rest and the produced dipole can interact with a nucleon in the target or projectile elastically or inelastically. Elastic interaction does not break the nucleon and does not lead to any energy loss. However, in inelastic interactions nucleons are broken, and due to hadronization, the remnants of the nucleon carry  $(1 - z_h)$  fraction of energy leading to  $z_h$  energy loss of the fast dipole. The produced hadrons do not interact since the density of the nucleons at fixed impact parameters are small in the nuclei (maximum 2 nucleons for gold). Finally, the sizes of produced dipoles during interaction with nuclei reaches the soft scale ( $1/\mu$ ) and due to hadronization the energy of the fastest hadron becomes  $z_h$  times smaller. Notice, that we assume for simplicity that both hadronization stages have the same  $z_h$ .

In our description of hadronization, we neglect the fragmentation process and instead for simplicity and clarity resort to the Local Parton-Hadron Duality (LPHD) principle [27, 28, 31, 55], namely we assume that the final-state hadronization is a soft process and cannot change the direction of the emitted radiation<sup>1</sup>. Then, the key question is to obtain the fraction of energy of the produced partonic system (in the pre-hadronization stage) which is carried away by hadron, the so-called  $z_{A,h}$  parameter. We have recently shown [27] that the data for the inclusive hadron production in  $pp$  collisions in a wide range of energies can be described if we assume that the energy loss in the process of fragmentation of the produced gluon into hadron reaches  $z_h \approx \frac{1}{2}$ . In the following section we calculate the value of  $z_A$  in the presence of a dense QCD medium within two different pre-hadronization pictures and we will show that both schemes give rather similar results.

### A. Quark-antiquark pre-hadronization model

As a first model, in spirit of all available hadronization models, we assume that hadronization passes a stage in which all gluons first go to quark-antiquark pairs (pre-hadronization)(see Fig. 2). Each pair then decays into hadron in the same way for hadron-hadron, hadron-nucleus and nucleus-nucleus interactions. In such a decay, the fastest hadron carries  $z_h$  fraction of energy of the pair.

In nucleus-nucleus interactions, the process of quark-antiquark pair creation is depicted in Fig. 3. In heavy ion collisions, the probability for a gluon<sup>2</sup> with an average transverse momentum  $Q_{s1}$  to decay into quark-antiquark ( $q\bar{q}$ )

<sup>1</sup> The same idea was used in the KLN [31] and the LR [28] approaches which describes the rapidity distribution of heavy-ion collisions data in a wide range of energies. This scheme also describes experimental data from  $e^+e^-$  annihilation into hadrons [55] and inclusive hadron productions in  $pp$  collisions including the recent LHC data [27].

<sup>2</sup> It is well known that at high energies the most produced partons are gluons. However, at RHIC energies the contribution of quarks, perhaps, are not negligible. However, in our approach similar to Ref. [36], we assume that gluons give the most important contribution at the highest RHIC energy as well. The main reasons behind this assumption are: the main contribution for the NMF factor measured at RHIC, stems from the momentum of the jet  $p_T = 8 - 20$  GeV for  $z = 1/2$ . The typical values of  $x = 2p_T/W/z$  turns out to be smaller than 0.1 for  $W = 200$  GeV while the quarks become essential at  $x \geq 0.2$ . Moreover, our main goal here is not to describe the experimental data but to answer the question whether it is possible to have the value of the NMF much smaller than 1 within the framework of the QCD factorization theorem.

pair at impact-parameter  $b$  with a relative transverse momentum denoted by  $\vec{k} \equiv \underline{k}$  ( $k^2 \equiv k^2$ ) has the following form<sup>3</sup> [56–58],

$$P_{AA}^{q\bar{q}}(y, b, k) = \frac{\alpha_s Q_{s1}^2}{8\pi^4} \int d^2r \int d^2r' e^{-i\frac{1}{2}\underline{k} \cdot (\underline{r} - \underline{r}')} \left\{ \frac{1}{2} \frac{\underline{r} \cdot \underline{r}'}{rr'} K_1(rQ_{s1}) K_1(r'Q_{s1}) + K_0(rQ_{s1}) K_0(r'Q_{s1}) \right\} \\ \times \frac{8C_F}{\pi^2\alpha_s} \left\{ \frac{1}{r^2} \left( 1 - e^{-\frac{1}{8}\underline{r}^2 Q_{s1}^2} \right) \left( 1 - e^{-\frac{1}{8}\underline{r}'^2 Q_{s2}^2} \right) + \frac{1}{r'^2} \left( 1 - e^{-\frac{1}{8}\underline{r}'^2 Q_{s1}^2} \right) \left( 1 - e^{-\frac{1}{8}\underline{r}^2 Q_{s2}^2} \right) \right. \\ \left. - \frac{1}{(\underline{r} - \underline{r}')^2} \left( 1 - e^{-\frac{1}{8}(\underline{r} - \underline{r}')^2 Q_{s1}^2} \right) \left( 1 - e^{-\frac{1}{8}(\underline{r} - \underline{r}')^2 Q_{s2}^2} \right) \right\}. \quad (4)$$

The factor in the first curly bracket is the distribution function  $\Psi(G \rightarrow q\bar{q}(r) \Psi^*(G \rightarrow q\bar{q}(r'))$  that describes the production of  $q\bar{q}$  dipoles by gluon with size  $r$  ( $r'$ ) in the amplitude and complex conjugated amplitude, respectively, before the passage through the nuclei (see Fig. 3). This gluon has typical transverse momentum  $Q_{s1}$ . The formula is written in the laboratory frame where the nucleus  $A_2$  is at rest. Eq. (4) is derived as a sum of all possible inelastic interactions. For example, for one inelastic interaction with nucleus  $A_2$  the contribution to Eq. (4) has the following form,

$$\int dz_1 e^{-\frac{1}{2}\sigma(Y, r) \rho z_1} \rho \sigma_{in} e^{-\frac{1}{2}\sigma(Y, r) \rho(2R_{A_2} - z_1)} e^{-\frac{1}{2}\sigma(Y, r) \rho 2R_{A_1}} e^{-\frac{1}{2}\sigma(Y, r') \rho(2R_{A_2} - z_1)} e^{-\frac{1}{2}\sigma(Y, r') \rho z_1}, \quad (5)$$

where  $\rho$  is the nucleon density and  $R_{A_i}$  is the radius of the nucleus  $A_i$ . Eq. (5) indicates that a dipole with a transverse size  $r$  interacts inelastically with the nucleon located at point  $x_1$  in nucleus  $A_2$  and exponents provide that no inelastic interactions occur with other nucleons in both nuclei in the amplitude. In the complex conjugate amplitude the same happens with a dipole with a transverse size  $r'$ . Note that for simplicity we assumed that the nuclear profile is cylindrical  $T(b) \approx 2R_A$  with  $b^2 \leq R_A^2$ .

The dipole-nucleon cross-section can be written as

$$\sigma(Y, r) = \frac{4\pi^2\alpha_s}{N_c} \int \frac{d^2l}{2\pi l^2} (1 - e^{i\underline{r} \cdot \underline{l}}) (1 - e^{-i\underline{r} \cdot \underline{l}}) \phi(Y, l^2) \xrightarrow{\underline{r} \cdot \underline{l} \ll 1} \frac{2\pi^2\alpha_s}{N_c} r^2 \int d^2\phi(Y, l^2), \quad (6)$$

where  $\phi$  is the unintegrated gluon density,  $Y$  is the rapidity of the dipole. In the right side of the above equation we have kept only the dominant contribution relevant at large transverse momentum of the produced dipole in the DGLAP approximation. Using Eq. (6), one can simplify Eq. (5) by the following replacement [33, 56]:

$$\sigma(Y, r) \rho T(b) \approx \sigma(Y, r^2) \rho 2R_A \rightarrow r^2 Q_{si}^2 / 8, \quad (7)$$

where  $Q_{si}$  is the gluon saturation momentum in the nuclei  $i$  and can also depend on the impact-parameter  $b$ . In the same fashion, for inelastic dipole-nucleon cross-section we have the following expression

$$\sigma_{in}(Y, r, r') = 2 \frac{4\pi^2\alpha_s}{N_c} \int \frac{d^2l}{2\pi l^2} (1 - e^{i\underline{r} \cdot \underline{l}}) (1 - e^{-i\underline{r}' \cdot \underline{l}}) \phi(Y, l^2) \xrightarrow{\underline{r} \cdot \underline{l} \ll 1} \frac{2\pi^2\alpha_s}{N_c} 2\underline{r} \cdot \underline{r}' \int d^2\phi(Y, l^2), \\ \sigma_{in}(Y, r, r') \rho 2R_A \rightarrow 2\underline{r} \cdot \underline{r}' Q_{si}^2 / 8. \quad (8)$$

The inelastic cross-section can be generally written in a form of convolution of two amplitudes  $\sigma_{in} = \sum_n A(r; \{k_i\}) \times A^*(r'; \{k_i\})$  with sum over produced  $n$ -partons with momenta  $k_i$ . Eq. (8) describes the inelastic cross-section for the process in which the dipole of size  $r$  in the amplitude interacts with nucleon while in the complex-conjugated amplitude, the dipole with size  $r'$  interacts with the same nucleon as in the amplitude.

Note that Eq. (4) integrated over the impact-parameter  $b$  gives the inclusive cross-section for  $q\bar{q}$ -pair with a relative transverse momentum  $k$ . It incorporates the first stage of the process, namely the parton production (shown as a dark blob in Fig. 2). Here, we assumed that the produced quark-antiquark pair is moving in the classical gluon field, neglecting the evolution in the rapidity interval  $y$ . We believe that this assumption will not be essential in estimating the energy loss.

In calculating the average energy loss, we assume that in each inelastic interaction, the dipole energy decreases  $z_h$  times due to the pre-hadronization process. For elastic interaction we also introduce factor  $z_{h,el}$  which describes the fraction of energy that is carried by the fastest hadron. For simplicity we assume that  $z_h = z_{h,el}$ . For elastic

---

<sup>3</sup> The derivation is based on the approaches suggested in Refs. [17, 33, 59–61].

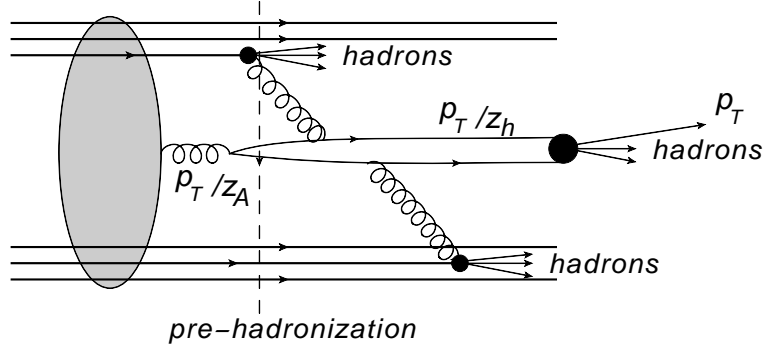


FIG. 2: Inclusive production of hadrons in nucleus-nucleus collision through the stage of pre-hadronization.

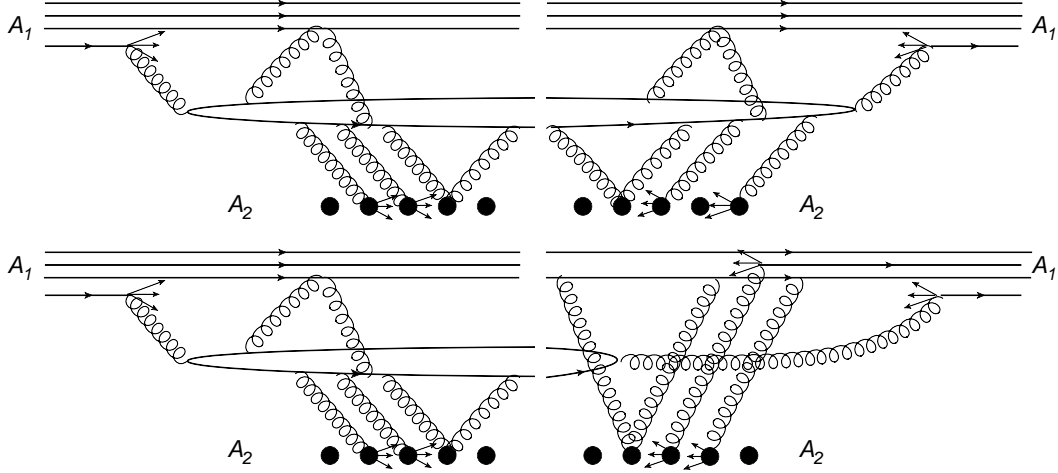


FIG. 3: Inclusive production of quark-antiquark pair (dipole) in nucleus-nucleus collisions.

scattering the energy loss emerges only on the last stage, namely quark-antiquark (or gluon) hadron transition. An additional source of the energy loss in nucleus collisions stems from the inelastic interaction of  $q\bar{q}$ -pairs (or gluons). In this process, soft gluons (slower than propagating  $q\bar{q}$ -pair or gluon) are produced. Then, these gluons produce hadrons due to hadronization and these hadrons carry  $(1 - z_h)$  energy of colliding gluon. Due to this process, the propagating gluon loses  $z_h$  fraction of energy per each inelastic interaction. Therefore, in order to calculate the energy loss in a nucleus we need to find the probability to have the inelastic collisions with  $i$  nucleons in a nucleus ( $P_i$ ) and multiply it by the energy loss in each such collisions. In other words,

$$\langle z_A \rangle = z_h P_0 + \sum_{i=1} z_h^i P_i, \quad (9)$$

where we define the probability  $P_i = \sigma_{i,in}/\sigma_{in}$  with  $\sigma_{i,in}$  being the contribution of inelastic  $i$  collisions to the total inelastic cross section  $\sigma_{in}$ . The latter is given by Eqs. (4,5). Therefore, in order to calculate the resulting average energy-loss of the dipole propagating through the medium, in Eqs. (4,5), one needs to find  $\sigma_{i,in}$  and multiply every inelastic collision by the factor  $z_h$ . From Eqs. (6,8) one can see that each factor  $\underline{r} \cdot \underline{r}'$  is associated with the inelastic interaction. Therefore, this multiplication can be carried out by the following replacement in Eq. (4),

$$(\underline{r} - \underline{r}')^2 = r^2 + r'^2 - 2\underline{r} \cdot \underline{r}' \rightarrow r^2 + r'^2 - 2z_h \underline{r} \cdot \underline{r}'. \quad (10)$$

The fact that factor  $\underline{r} \cdot \underline{r}'$  is associated with the inelastic cross-section in Eq. (8) for  $P_{AA}^{q\bar{q}}(y, b, k)$  can be understood on general grounds. For example, the inelastic dipole cross-section at two-gluon exchange level is  $\sigma_{in} \propto rr'$ . Notice that all other terms in expansion of  $P_{AA}^{q\bar{q}}(y, b, k)$  are of the order of  $r^2$  and  $r'^2$ .

The average energy loss can be then obtained from the following expression

$$\langle z_h \rangle_A \equiv z_A = \left( (z_h - 1) P_{AA}^{q\bar{q}}(y, b, k; z_h = 0) + P_{AA}^{q\bar{q}}(y, b, k; z_h) \right) / P_{AA}^{q\bar{q}}(y, b, k; z_h = 1). \quad (11)$$

In the above equation, the term  $P_{AA}^{q\bar{q}}(y, b, k; z_h = 0)$  describes the process of diffraction dissociation in which dipoles scatter only elastically and the term  $P_{AA}^{q\bar{q}}(y, b, k; z_h) - P_{AA}^{q\bar{q}}(y, b, k; z_h = 0)$  corresponds to the contribution of all inelastic processes with additional factor  $z_h$  for each rescattering.

Therefore, the first term in Eq. (11) only takes into account the processes of hadronization of the quark-antiquark with rapidity  $y$  that passes through the medium without an inelastic interaction. The second term in Eq. (11) is responsible for hadrons production in the entire kinematic region of rapidity  $y$ . Note that Eq. (11) is based on the assumption that if no energy-loss occurs in a single interaction  $z_h = 1$  then no energy-loss will follow in multiple-interactions namely  $z_A = 1$ , and if the parton loses all its energy in a single interaction  $z_h = 0$  then naturally in multiple-interactions we have also  $z_A = 0$ . The NMF Eq. (3) incorporating the pre-hadronization energy-loss effect, can be then rewritten in the following form,

$$\text{NMF} = \frac{1}{A^2} \frac{S_A^2}{S_p^2} \frac{\mathcal{T}(p_T / (z_A Q_{s,A}))}{\mathcal{T}(p_T / (z_h Q_{s,N}))}. \quad (12)$$

In Eq. (12) we use the fact that the transverse momentum of the emitted gluon in  $pp$  or  $AA$  collisions is equal to [27],

$$p_T^2(\text{gluon}) = p_T^2(\text{hadron}) / z_{h,A}^2 + k_T^2(\text{intrinsic}), \quad (13)$$

where  $k_T(\text{intrinsic})$  is the average intrinsic transverse momentum of the gluonic jet and since its value is small, it can be neglected.

## B. Gluon pre-hadronization model

In the second model, we consider a case that the produced gluon decays into hadrons after propagating through the nucleus (see Fig. 4).

Using the approaches suggested in Refs. [17, 59, 60] and summing all gluon exchange diagrams shown in Fig. 5 and neglecting again the small- $x$  evolution in the rapidity interval between the produced gluon and the target, the probability of gluon production in  $AA$  collisions with a transverse momentum  $k$  can be written in the following form,

$$\begin{aligned} P_{AA}^G(y, b, k) &= \frac{\alpha_s Q_{s1}^2}{8\pi^4} \int d^2r \int d^2r' e^{-i\frac{1}{2}\underline{k} \cdot (\underline{r} - \underline{r}')} \left\{ K_2(kr) K_2(kr') J_0(kr) J_0(kr') \right\} \\ &\times \frac{8C_F}{\pi^2\alpha_s} \left\{ \frac{1}{r^2} \left( 1 - e^{-\frac{1}{4}r^2 Q_{s1}^2} \right) \left( 1 - e^{-\frac{1}{4}r'^2 Q_{s2}^2} \right) + \frac{1}{r'^2} \left( 1 - e^{-\frac{1}{4}r'^2 Q_{s1}^2} \right) \left( 1 - e^{-\frac{1}{4}r^2 Q_{s2}^2} \right) \right. \\ &\quad \left. - \frac{1}{(\underline{r} - \underline{r}')^2} \left( 1 - e^{-\frac{1}{4}(\underline{r} - \underline{r}')^2 Q_{s1}^2} \right) \left( 1 - e^{-\frac{1}{4}(\underline{r} - \underline{r}')^2 Q_{s2}^2} \right) \right\}. \end{aligned} \quad (14)$$

In the derivation of the above equation, we assume that the gluon with transverse momentum  $k$  is emitted by the dipole with transverse size smaller than  $1/k$ . Indeed, all rescattering of the gluons emitted by dipoles with larger sizes are already included in Eq. (2) [17]. In this case the emission of gluon from interaction with a nucleon can be considered as a decay of the nucleon to two colorless dipoles. This can be better conceived at large  $N_c$  limit where an adjoint (gluon) dipole can be decomposed into two fundamental (quark) dipoles. These two dipoles penetrate through the medium with the same cross section. The wave function of such a gluon has been calculated in Ref. [62]. Therefore, this approach differs from the quark-antiquark pre-hadronization model mainly by the value of the dipole cross section:  $\sigma_G(Y, r^2) \approx 2\sigma_{dipole}$ . In other words, the main differences between Eq. (14) and Eq. (4) is that in Eq. (14) the value of the saturation momentum is two times larger than in Eq. (4).

Notice than in general, we are dealing here with rescatterings of two dipoles with different sizes. For example, in the dipole-nucleus scattering amplitude at the first stage of the process, the incoming dipole with size  $x_{12} = |\vec{r}_1 - \vec{r}_2|$  emits a gluon with a coordinate  $\vec{r}$  and this process at  $N_c \gg 1$  can be considered as a decay of the dipole  $x_{12}$  to two dipoles  $x_{13} = |\vec{r}_1 - \vec{r}|$  and  $x_{23} = |\vec{r}_2 - \vec{r}|$ . These two dipoles take part in interactions with a nucleus. However, in the inclusive cross-section we integrate over the momenta that carry quark (antiquark) with coordinates  $r_1$  and  $r_2$ . Therefore, this cross-section does not depend on these coordinates and both can be put equal to zero. That is the reason why the inclusive cross-section here can be viewed as a propagation of dipole with size  $r$  in the amplitude and with size  $r'$  in the complex conjugated one.

In order to calculate the average energy loss of the produced gluon at the pre-hadronization stage  $\langle z_h \rangle_A$ , we again follow the steps in Eqs. (10,11) by replacing  $P_{AA}^{q\bar{q}} \rightarrow P_{AA}^G$ . The corresponding NMF can be then obtained by Eq. (12).

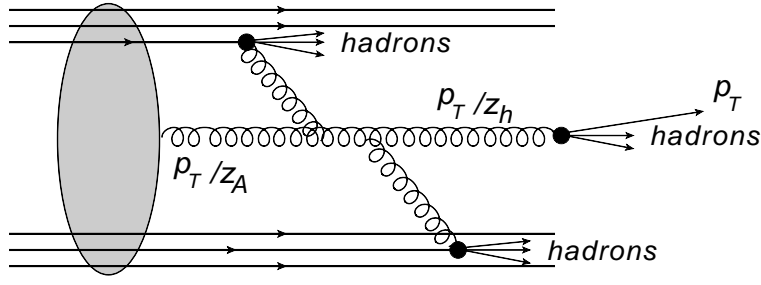


FIG. 4: Inclusive production of hadrons in nucleus-nucleus collision due to gluon decay.

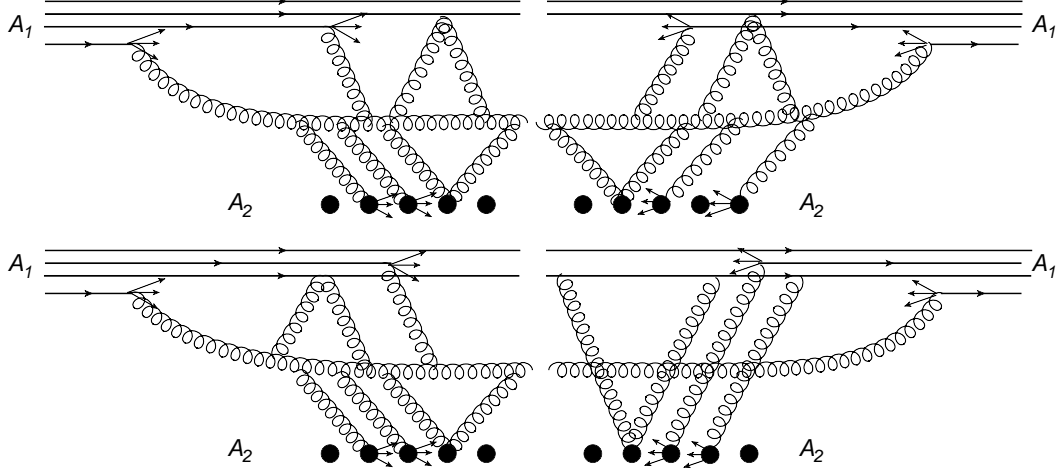


FIG. 5: The passage of the produced gluon in nucleus-nucleus collision.

It is worthwhile mentioning that in Eq. (12) we take into account both the first stage, namely  $q\bar{q}$ -pair (or gluon) production (the dark blobs in Figs. 2, 4), and also the energy-loss effect due to the passage of the produced  $q\bar{q}$ -pair (or gluon) through the medium (see Figs. 3, 5). In a sense, we unfolded the  $k_T$  factorization and made the corresponding corrections due to the pre-hadronization energy-loss effect since the hadronization leads to its violation.

### III. LARGE $p_T$ BEHAVIOR OF NMF

At very large  $p_T$ , the pQCD factorization theorem [7–11] can be used. The production of gluon-jet with transverse momentum  $p_T$  (see Fig. 6) in nucleus-nucleus collisions can be simply written as

$$\begin{aligned} \frac{d\sigma_{AA}}{dy, d^2p_T}|_{y=0} &= A^2 \frac{d\sigma_{pp}}{dy, d^2p_T} = \sigma_{\text{hard}} x_1 G_A(x_1 = 2p_T/\sqrt{s}, p_T) x_2 G_A(x_2 = 2p_T/\sqrt{s}, -p_T), \\ &= A^2 \frac{\alpha_s^2(p_T)}{p_T^4} x_1 G_p(x_1 = 2p_T/\sqrt{s}, p_T) x_2 G_p(x_2 = 2p_T/\sqrt{s}, -p_T) \rightarrow A^2 \frac{\alpha_s^2(p_T)}{p_T^4} (p_T^2/Q_0^2)^{2\gamma}, \end{aligned} \quad (15)$$

where  $xG_P$  (or  $xG_A$ ) is the gluon structure function for the proton (or nucleus) and  $Q_0$  is a separation scale so as for  $p_T > Q_0$  one can use perturbative QCD. In the above expression,  $\sigma_{\text{hard}}$  is the perturbative partonic cross section computable up to a given order in  $\alpha_s$ , and  $\gamma$  denotes the anomalous dimension. It is well-known that in the leading-order for  $x \rightarrow 1$  we have  $\gamma \rightarrow \alpha_s$ . Therefore, at very large  $p_T \gg Q_s$  but  $p_T \ll \sqrt{s}$ , the NMF obtained via Eqs. (1,13,15) is equal to

$$\text{NMF} \xrightarrow{\sqrt{s} \gg p_T \gg Q_s} \frac{\alpha_s^2(p_T/z_A) (p_T/z_A)^{4\alpha_s(p_T/z_A)}}{\alpha_s^2(p_T/z_h) (p_T/z_h)^{4\alpha_s(p_T/z_h)}} \times \left(\frac{z_A}{z_h}\right)^4. \quad (16)$$

The value of the NMF can be then immediately estimated at large  $p_T$  via Eq. (16) by only knowing the value of  $z_A$ ,  $z_h$  and also  $\Lambda_{QCD}$  which appears in the running strong-coupling. For the running coupling  $\alpha_s$ , we employ the scheme

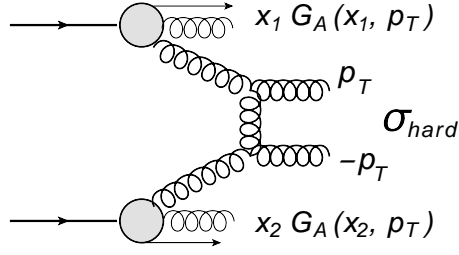


FIG. 6: The gluon production accordingly to the factorization theorem at large  $p_T$ .

used in Ref. [27] at the leading-order. We postpone the numerical discussion to the next section, and in the rest of this section, we address another important question, namely at which  $p_T$  the expression given in Eq. (16) can be used. Let us for sake of simplicity work in double log approximation with the anomalous dimension  $\gamma = \bar{\alpha}_s/\omega$  where  $\omega$  is Mellin image of  $\ln(1/x)$  and  $\bar{\alpha}_s = \alpha_s N_c/\pi$ . In such an approximation, the nuclear gluon structure function up to a numerical constant  $\mathcal{K}$  can be written as [63]

$$\begin{aligned} \frac{x G_A(x = 2p_T/\sqrt{s}, p_T)}{p_T^2} &= \mathcal{K} \exp \left( \sqrt{4\bar{\alpha}_s \ln(1/x) \ln(p_T^2/Q_0^2)} - \ln(p_T^2/Q_0^2) + l_A \right), \\ &= \mathcal{K} \exp \left( \sqrt{L \ln(p_T^2/Q_0^2)} - \ln(p_T^2/Q_0^2) + l_A \right), \end{aligned} \quad (17)$$

where  $L = 4\bar{\alpha}_s \ln(1/x) = \ln(Q_s^2(A; x)/Q_s^2(A; x = x_0))$  and  $l_A = (1/3) \ln A$ . The equation for the saturation scale has the form [63],

$$L l_s = (l_s - l_A)^2, \quad (18)$$

with a solution,

$$l_s(L, l_A) \equiv \ln(Q_s^2(A; x)/Q_0^2) = \frac{L}{2} + l_A \sqrt{\left(\frac{L}{2}\right)^2 + L l_A}. \quad (19)$$

One can see that  $G_A$  given by Eq. (17) at  $Q^2 = Q_0^2$  is proportional to  $A^{1/3}$ . Introducing a new variable  $\mathcal{Z} = \ln(p_T^2/Q_s^2(A; x))$  one can rewrite Eq. (17) in the following form

$$\frac{x G_A(x = 2p_T/\sqrt{s}, p_T)}{p_T^2} = \mathcal{K} \exp(H(\mathcal{Z}, L, l_A)), \quad (20)$$

with a notation,

$$H(\mathcal{Z}, L, l_A) = \sqrt{L(\mathcal{Z} + l_s(L, l_A))} - \mathcal{Z} - l_s(L, l_A) + l_A. \quad (21)$$

Expanding  $H(z, L, l_A)$  at small  $z$ , we have

$$H(\mathcal{Z}, L, l_A) = -h_1(L, l_A) \mathcal{Z} + h_2(L, l_A) \mathcal{Z}^2 + \mathcal{O}(\mathcal{Z}^3), \quad (22)$$

with

$$h_1(L, l_A) = 1 - \frac{L}{2\sqrt{\frac{L^2}{2} + L l_A + L \sqrt{l_A^2 + \left(\frac{L}{2} + l_A\right)^2}}}, \quad (23)$$

$$h_2(L, l_A) = \frac{L^2}{2\sqrt{2} \left( L \left( L + 2l_A + \sqrt{L^2 + 4L l_A + 8l_A^2} \right) \right)^{3/2}}. \quad (24)$$

The function  $h_1$  and  $h_2$  versus  $L$  for the gold are shown in Fig. 7. It is seen that at large values of  $L$ , we have  $h_1 \rightarrow 1/2$  and  $h_2 \rightarrow 1/8L$  in accordance with finding in Ref. [30]. Using the KLN saturation model parametrization, the value of  $L$  is about  $L \approx 1$  and  $1.7$  for RHIC and the LHC energies, respectively [31]. The condition [30]

$$h_2 \mathcal{Z}^2 > 1, \quad (25)$$



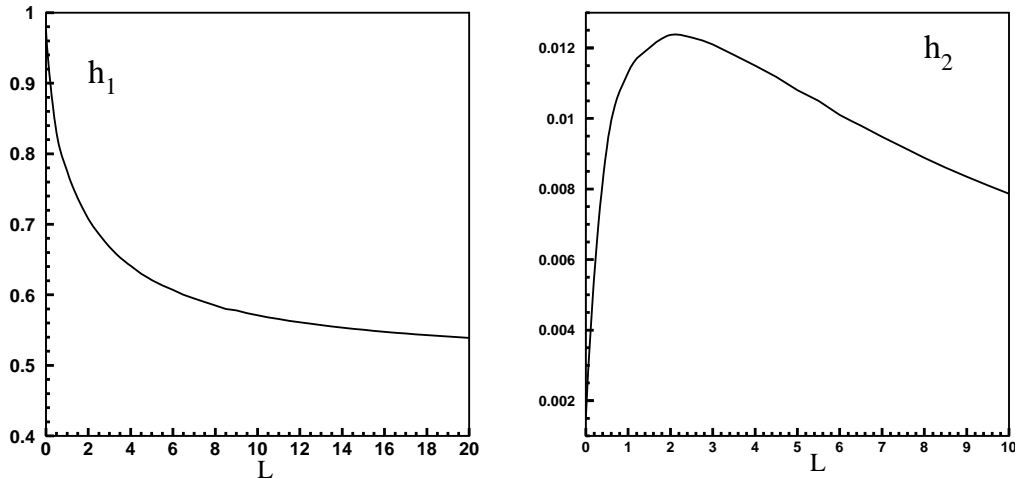


FIG. 7: Function  $h_1$  and  $h_2$  defined in Eqs. (23,24) versus  $L$ .

characterizes that  $\mathcal{Z}$  is large and the scattering amplitude is far away from the saturation domain, out of the region where we have the geometric scaling behavior at large values of  $L$ . Indeed, the first term in Eq. (22) leads to the geometric scaling behavior at large values of  $L$  while the second term violates this behavior even at large  $L$ . Therefore, the condition given in Eq. (25) ensures that the gluon structure function defined in Eq. (20) gives the perturbative form used in Eq. (15).

Eq. (25) leads to  $p_T > 40$  GeV for the RHIC energies and to  $p_T > 60$  GeV for the LHC energies for the produced gluon. In terms of the transverse momenta of the produced hadron we expect that for  $p_T \geq 40$   $z_A \approx 15$  GeV at the RHIC and for  $p_T \geq 60$   $z_A \approx 25$  GeV at the LHC, the NMF defined in Eq. (16) to be reliable. Notice that if the pre-hadronization stage in  $AA$  and  $pp$  collisions was the same, namely  $z_A = z_h$ , then the NMF defined in Eq. (16) was identically equal to one  $R_{AA} = 1$ . Therefore, the main source of suppression of the NMF for hadrons at high- $p_T$  is due to further energy loss of the produced gluons in  $AA$  collisions compared to  $pp$  collisions at the pre-hadronization stage, namely  $z_A < z_p$ . Notice that on average  $z_A$  and  $z_p$  vary slowly with  $p_T$  for the kinematic region of our interest here [28]. Note also that the  $p_T$ -dependence in Eq. (16) mainly enters through the running of the strong-coupling. This justifies the flatness of the NMF  $R_{AA}$  over large range of  $p_T$ . In the next section we show that the numerical value obtained for  $z_A$  from the pre-hadronization mechanisms introduced in section II can indeed describe the observed suppression of hadrons at large- $p_T$  at RHIC. For smaller values of  $p_T$ , Eq. (2) may be used with the dipole amplitude  $N$  being in the region where we still have the geometric scaling behavior for  $N \equiv N(z)$  [30].

#### IV. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENTAL DATA

In order to estimate the value of  $\langle z_h \rangle_A$ , we use the KLN saturation parametrization [31] with  $Q_s = 1.41$  GeV at  $\sqrt{s} = 200$  GeV for gold and we take  $k = 2$  GeV as a typical scale for soft interaction. The chosen soft interaction scale corresponds to the value of the soft Pomeron slope  $k \approx 1/\sqrt{\alpha'_P}$  with  $\alpha'_P = 0.25$  GeV<sup>-2</sup> [60, 61, 64]. It turns out that both pre-hadronization models give very close values for  $\langle z_h \rangle_A/z_h$ : 0.766 and 0.744 for the quark-antiquark and the gluon pre-hadronization model respectively, at the RHIC energy  $\sqrt{s} = 200$  GeV. The value of  $z_h \approx 1/2$  was obtained from a fit to the experimental data in  $pp$  collisions for the average transverse momentum of charged hadrons for a wide range of energies [27].

Again using the KLN parametrization for the nuclear saturation scale, we estimated the value of  $\langle z_h \rangle_A/z_h$  at the LHC energies, assuming  $z_h \approx 1/2$ . At higher energies or larger value of the saturation scale  $Q_s$ , one can approximate the integrand of Eqs. (4,14) to its maximum value at  $r = r' \approx 1/Q_s$ . This leads to a similar value for  $z_A$  in both pre-hadronization models at higher energies, since the over-all factors in Eqs. (4,14) will be canceled out via Eq. (11). Indeed, full numerical solution is in accordance with this observation and we found that both pre-hadronization models result to a very similar value for  $z_A$  at the LHC energies, namely  $\langle z_h \rangle_A/z_h \approx 0.82, 0.795$  at  $\sqrt{s} = 5.5, 2.75$  TeV, respectively.

Our results for the NMF for 0–10% centrality in  $Au+Au$  collisions using the  $k_t$  factorization Eq. (12) supplemented with solutions of the BK and the modified BK equation that preserves the energy conservation [65] is shown in Fig. 8.

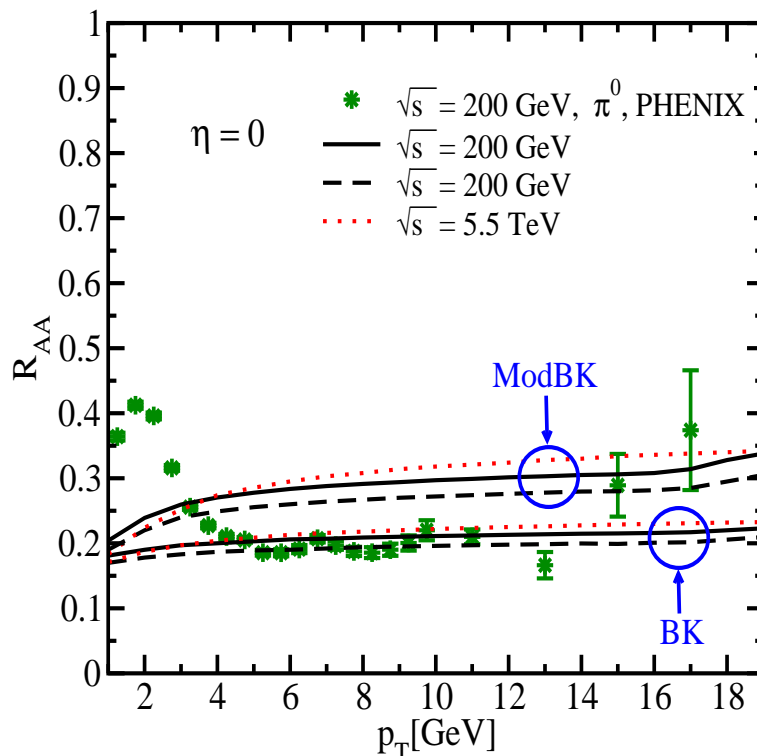


FIG. 8: The NMF for hadrons obtained by Eq. (12) for 0 – 10% centrality  $Au + Au$  collisions at various energies. Function  $\mathcal{T}(x_\perp)$  is calculated using the solution of the BK and the modified BK equations. The solid and dashed lines correspond to the quark-antiquark and gluon pre-hadronization models, respectively. The experimental data is taken from Ref. [1].

In the modified BK equation (denoted by ModBK in Fig. 8), the next-to-leading order corrections to the BFKL kernel are taken into account which leads to conservation of energy in the framework of non-linear equation [66]. We recall that here we resort to the LPHD principle for the final state hadronization. But the energy-loss effect in the pre-hadronization both in  $pp$  and  $AA$  collisions are effectively incorporated in the results shown in Fig. 8. We believe that the difference between two pre-hadronization models (introduced in Sec. II) in  $AA$  collisions gives reasonable estimates of the errors which are unavoidable in the situation that we do not know the theory of pre-hadronization in a dense medium. As we already pointed out, the differences between these two pre-hadronization models at LHC is negligible. The energy-loss effect reduces the NMF defined in Eq. (12) about  $30 \div 50\%$ . Notice that this reduction does not depend on a given BK solution and incorporates the missing energy-loss effect into the  $k_T$  factorization.

We should stress that the numerical estimates in Fig. 8 was obtained from the BK equation with a simplified kernel [63, 65]. Such estimates is less reliable for describing the experimental data, even though, interesting enough, such a rough approximation apparently is in agreement with RHIC data at high- $p_T$  within the errors. Notice that in fact the validity of the  $k_T$  factorization Eq. (12) beyond the extended geometric-scaling region at high- $p_T$  is questionable, see also Sec. I. An important message here is that the smallness and the high- $p_T$  flatness of the NMF obtained from Eq. (12) and the BK equation is rather generic and it does not strongly depend on the details of the BK solutions in our interested kinematic region here, see also recent interesting paper by Albacete and Marquet [67]. It is also seen from Fig. 8 that as one may expect, the onset of the flatness of the NMF at the LHC energy occurs at higher  $p_T$ . Fig. 8 indicates that the energy-conservation constrain on the BK equation which is important at higher  $p_T$  tends to enhance the NMF. This is due to the fact that the ModBK solution correctly incorporates the behavior of the anomalous dimension at large Bjorken- $x$  by matching to the DGLAP value.

In order to make more reliable prediction for the NMF for the neutral pion  $\pi^0$ , we next re-calculate the NMF from Eq. (12) by using the impact-parameter dependent CGC saturation model in  $pp$  [27] and  $AA$  collision [28]. It has been already shown that such a scheme describes HERA data at small- $x$  [68] and the hadron multiplicities in nucleus-nucleus collisions at RHIC [28] and provided correct predictions [27] for the inclusive hadron production at the LHC. The importance of the inclusion of the impact-parameter dependence in the saturation models and the BK equation has been addressed in Refs. [27, 28, 69]. Our results at RHIC and the LHC energies in 0 – 10% centrality  $Au + Au$  (at RHIC) and  $Pb + Pb$  (at the LHC) collisions, is shown in Fig. 9. We recall that Eq. (12) has  $x_\perp$ -scaling property which is valid only at low transverse momentum roughly for  $p \leq 3 \div 4Q_s$ . In the saturation model [68] used

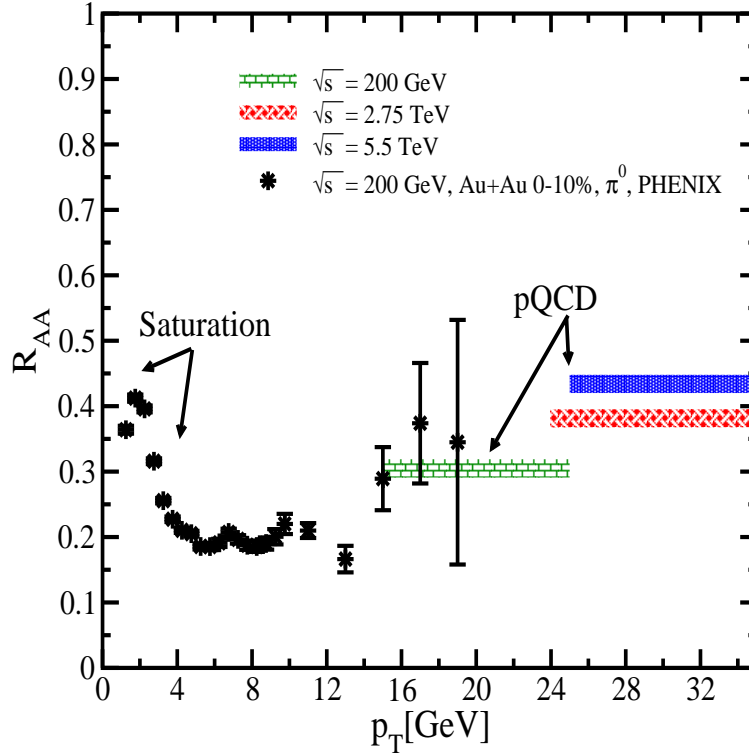


FIG. 9: The NMF for hadrons obtained by Eq. (12) at low  $p_T$  and Eq. (16) at high  $p_T$  for 0 – 10% centrality  $AA$  collisions at various energies. Function  $\mathcal{T}(x_\perp)$  is calculated within the saturation (CGC) approach [27, 28]. We also show pQCD results obtained in Sec. III. The experimental data is taken from Ref. [1].

in Fig. 9, the saturation scale varies very slowly with energy and for  $p_T = 1$  GeV at midrapidity for central collisions we have  $Q_s = 0.76$  GeV for proton at  $\sqrt{s} = 5.5$  TeV. Therefore, the upper limit of  $x_\perp$ -scaling behavior in this model is about  $p_T \approx 3$  GeV for  $pp$  collisions (the reference for the NMF) at  $\sqrt{s} = 5.5$  TeV at midrapidity. We therefore, only show the CGC results coming from Eq. (12) for  $p_T < 4$  GeV. Here, we only concentrate on the CGC results at rather low- $p_T$  where we can also give a good description of spectra in  $pp$  collisions [27]. At the RHIC energy  $\sqrt{s} = 200$  GeV at mid-rapidity, for  $p_T > 2$  GeV, we have  $x > 0.01$  which is beyond the validity of the CGC prescription. Note also that the dipole model used here was obtained from a fit to HERA data for virtuality  $Q^2 \in [0.25, 45]$  GeV<sup>2</sup>. Therefore, this model is valid for  $p_T \in [0.7, 6.7]$  GeV. Again, we employ the LPHD principle to connect pre-hadronization to final hadronization stage. This leads to two different over-all factors for the spectra in  $pp$  and  $AA$  collisions which are fixed with the experimental data at lower energies [27, 28]. The appearance of two different over-all factors for the normalization is partly due to the fact that pre-hadronization leads to different effective masses for the mini-jet in  $AA$  and  $pp$  collisions [27, 28]. It is seen from Fig. 9 that our results without any free-parameter to adjust, is in accordance with the PHENIX data at low- $p_T$ , in contrast to the impact-parameter independent BK solution shown in Fig. 9 with a zero mini-jet mass. At low- $p_T$ , the energy-loss in medium is less important and we have taken  $z_A = z_h = 1/2$  for  $p_T < 4$  GeV. Notice that the actual value of  $R_{AA}$  at low  $p_T$  calculated via the  $k_T$  factorization may be subjected to about 15 ÷ 25% errors due to the uncertainties in the hadron multiplicity obtained within the same framework in  $AA$  collisions. Moreover, here the contribution of quarks was neglected which is perhaps less justifiable at midrapidity at low-energy.

At high- $p_T$ , we rely on the improved pQCD factorization result given in Eq. (16) (also shown in Fig. 9). The only external parameter in the NMF defined in Eq. (16) is the parameter  $z_A$  which incorporates the energy-loss effect at the pre-hadronization stage. The value of the parameter  $z_A$  was already calculated in Sec. II (see also above) at RHIC and the LHC energies. As we already argued Eq. (16) is valid at high transverse momentum of the produced hadrons for  $p_T \geq 40$   $z_A \approx 15$  GeV at RHIC and for  $p_T \geq 60$   $z_A \approx 25$  GeV at the LHC energies. Our predictions for the NMF suppression and the onset of the flatness of the NMF at high- $p_T$  both agree with the PHENIX data. As we already pointed out Eq. (16) naturally leads to the flatness of the NMF at high  $p_T$  since  $p_T$  dependence mainly enters via the running strong-coupling and varies very slowly with  $p_T$ . In Fig. 9, we also show our predictions both at low and high  $p_T$  at the LHC energies  $\sqrt{s} = 2.75$  and 5.5 TeV. The uncertainties in our formulation is less than 10% which come mainly from the normalization in Eq. (12). The band in Fig. 9 indicates about 2% theoretical error

which also includes the discrepancies between two pre-hadronization schemes. The predictions of other approaches at the LHC can be found in Ref. [70].

## V. CONCLUSIONS

In this paper first we developed a simple picture for calculating the energy-loss effect. In our approach, the energy-loss effect stems from the process of the pre-hadronization. We showed that the estimates of the energy-loss in two different models of the pre-hadronization lead to very close values and is able to explain the measured  $R_{AA}$  at high transverse momentum of produced hadrons at RHIC. We showed that the small value of the nuclear modification factor at high- $p_T$  is mainly due to the energy-loss effect in the pre-hadronization stage.

We also investigated the NMF obtained from the  $k_T$  factorization supplemented with solutions of the BK and the modified BK equation that preserves the energy conservation. We showed that the smallness ( $R_{AA} < 0.5$ ) and the high- $p_T$  flatness of the NMF obtained from Eq. (12) and the BK equation is rather generic and it does not strongly depend on the details of the BK solutions. We showed that the modified BK solution which includes the energy conservation tends to slightly enhance the NMF at high- $p_T$ . This is due to the fact that the modified BK equation properly includes the anomalous dimension at  $x \approx 1$  which coincides with the anomalous dimension of the DGLAP equation while the BK equation only reproduces its double log limit. This indicates that one should be cautious as extrapolating the  $k_T$  factorization results to higher  $p_T$  is less reliable if the running anomalous dimension obtained from the BK solution does not match to the DGLAP value at high- $p_T$ .

We showed that at high- $p_T$ , the pQCD factorization incorporated with the pre-hadronization energy-loss effect Eq. (16) results naturally to flatness of the NMF. We obtained the onset of the applicability of Eq. (16) to be for  $p_T > 40$  GeV at RHIC and for  $p_T > 60$  GeV at the LHC for the produced gluon. Notice that in this kinematic region the NMF for the produced jet should approach to one  $R_{AA}^g \rightarrow 1$ . However, the NMF of the produced hadron is very small  $R_{AA}^{\pi^0} < 0.5$  due to the pre-hadronization effect, namely  $z_A/z_p < 1$ . In terms of the transverse momenta of the produced hadron we expect that this to be for  $p_T \geq 15$  GeV at RHIC and for  $p_T \geq 25$  GeV at the LHC. As one can see from Figs. 8, 9 the hadronization model is responsible for  $30 \div 50\%$  of the NMF value while  $50 \div 70\%$  of this value stems from the BK equation and  $k_T$  factorization.

One of important check of our hadronization model will be the estimates of the NMF in deuteron-nucleus collisions which will be published elsewhere. Here we would like only to mention that in our approach, estimates for the NMF in proton-gold collision at the RHIC energy  $\sqrt{s} = 200$  GeV at midrapidity gives only upto 10% contribution due to the hadronization leading to the value of the NMF about  $0.95 \pm 0.05$ .

Finally, it is worth mentioning that the main assumption that in the process of hadronization the fastest particle carries the final part of the initial energy is a typical feature of all hadronic reactions as well as the production of hadrons in DIS.

## Acknowledgments

This work was supported in part by Conicyt Programa Bicentenario PSD-91-2006 and the Fondecyt (Chile) grants 1090312 and 1100648.

- 
- [1] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. **C82** (2010) 011902 [arXiv:1005.4916].
  - [2] I. Arsene *et al.* [BRAHMS Collaboration], Phys. Rev. Lett. **91** (2003) 07305.
  - [3] I. Arsene *et al.* [BRAHMS Collaboration], Phys. Rev. Lett. **93** (2004) 242303 [nucl-ex/0403005].
  - [4] I. Arsene *et al.* [BRAHMS Collaboration], Phys. Lett. **B650** (2007) 219 [nucl-ex/0610021].
  - [5] S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **96** (2006) 032301 [nucl-ex/05100].
  - [6] M. J. Tannenbaum, “Critical examination of RHIC paradigms-mostly high  $p_T$ ”, Workshop on Critical Examination of RHIC Paradigms, Austin, TX, April 14-17 (2010), arXiv:1008.1536.
  - [7] J. C. Collins, D. E. Soper and G. F. Sterman, Adv. Ser. Direct. High Energy Phys. **5** (1988) 1 [hep-ph/0409313].
  - [8] J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. **B308** (1988) 833.
  - [9] G. T. Bodwin, Phys. Rev. **D31** (1985) 2616; Phys. Rev. **D34** (1986) 3932.
  - [10] J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. **B261** (1985) 104.
  - [11] X-N. Wang, Phys. Rev. **C61** (2000) 064910; for a review see: J. F. Owens, Rev. Mod. Phys. **59** (1987) 465.
  - [12] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rep. **100** (1983) 1.
  - [13] A. H. Mueller and J. Qiu, Nucl. Phys. **B268** (1986) 427; J.-P. Blaizot and A. H. Mueller, Nucl. Phys. **B289** (1987) 847.

- [14] L. McLerran and R. Venugopalan, Phys. Rev. **D49** (1994) 2233; **D49** (1994) 3352; **D50**, (1994) 2225; **D53** (1996) 458; **D59** (1999) 094002.
- [15] I. Balitsky, Nucl. Phys. **B463** (1996) 99; Y. Kovchegov, Phys. Rev. **D60** (2000) 034008.
- [16] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Phys. Rev. **D59** (1998) 014014 [hep-ph/9706377]; Nucl. Phys. **B504** (1997) 415 [hep-ph/9701284]; E. Iancu, A. Leonidov and L. D. McLerran, Phys. Lett. **B510** (2001) 133 [hep-ph/0102009]; Nucl. Phys. **A692** (2001) 583 [hep-ph/0011241]; H. Weigert, Nucl. Phys. **A703** (2002) 823 [hep-ph/0004044].
- [17] Y. V. Kovchegov and K. Tuchin, Phys. Rev. **D65** (2002) 074026 [hep-ph/0111362].
- [18] M. A. Braun, Eur. Phys. J. **C48** (2006) 501 [hep-ph/0603060].
- [19] M. A. Braun, Phys. Lett. **B483** (2000) 105 [hep-ph/0003003].
- [20] C. Marquet, Nucl. Phys. **B705** (2005) 319 [hep-ph/0409023].
- [21] A. Kovner and M. Lublinsky, JHEP **0611** (2006) 083 [hep-ph/0609227].
- [22] E. Levin and A. Prygarin, Phys. Rev. **C78** (2008) 065202 [arXiv:0804.4747].
- [23] S. Catani, M. Ciafaloni and F. Hautmann, Nucl. Phys. **B366** (1991) 135.
- [24] S. Catani, M. Ciafaloni and F. Hautmann, Nucl. Phys. Proc. Suppl. **A29** (1992) 182.
- [25] J. C. Collins and R. K. Ellis, Nucl. Phys. **B360** (1991) 3.
- [26] E. M. Levin, M. G. Ryskin, Yu. M. Shabelski and A. G. Shuvaev, Sov. J. Nucl. Phys. **53** (1991) 657 [Yad. Fiz. **53** (1991) 1059].
- [27] E. Levin and A. H. Rezaeian, Phys. Rev. **D82** (2010) 014022 [arXiv:1005.0631].
- [28] E. Levin and A. H. Rezaeian, Phys. Rev. **D82** (2010) 054003 [arXiv:1007.2430].
- [29] J. Bartels and E. Levin, Nucl. Phys. **B387** (1992) 617; A. M. Stasto, K. J. Golec-Biernat, and J. Kwiecinski, Phys. Rev. Lett. **86** (2001) 596.
- [30] E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. **A708** (2002) 327 [hep-ph/0203137].
- [31] D. Kharzeev and M. Nardi, Phys. Lett. **B507** (2001) 121; D. Kharzeev and E. Levin, Phys. Lett. **B523** (2001) 79; D. Kharzeev, E. Levin and M. Nardi, Phys. Rev. **C71** (2005) 054903; D. Kharzeev, E. Levin and L. McLerran, Phys. Lett. **B561** (2003) 93.
- [32] B. Z. Kopeliovich, A. H. Rezaeian, H. J. Pirner and I. Schmidt, Phys. Lett. **B653** (2007) 210 [arXiv:0704.0642]; arXiv:0707.2040; B. Z. Kopeliovich, H. J. Pirner, A.H. Rezaeian and I. Schmidt, Phys. Rev. **D77** (2008) 034011 [arXiv:0711.3010]; B. Z. Kopeliovich, A. H. Rezaeian and I. Schmidt, Nucl. Phys. **A807** (2008) 61 [arXiv:0712.2829]; B. Z. Kopeliovich, E. Levin, A. H. Rezaeian and I. Schmidt, Phys. Lett. **B675** (2009) 190 [arXiv:0902.4287].
- [33] A. H. Rezaeian and A. Schäfer, Phys. Rev. **D81** (2010) 114032 [arXiv:0908.3695].
- [34] S. Turbide, C. Gale, E. Frodermann and U. Heinz, Phys. Rev. **C77** (2008) 024909.
- [35] A. H. Mueller, Phys. Lett. **B668** (2008) 11 [arXiv:0805.3140].
- [36] D. Kharzeev, Y. V. Kovchegov and K. Tuchin, Phys. Lett. **B599** (2004) 23-31. [hep-ph/0405045].
- [37] B. Z. Kopeliovich, arXiv:1007.4513.
- [38] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **98** (2007) 232301 [nucl-ex/0611020].
- [39] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **101** (2008) 122301 [arXiv:0801.0220].
- [40] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. **C80** (2009) 041902 [arXiv:0904.0439].
- [41] R. Baier, A. H. Mueller, D. T. Son and D. Schiff, Nucl. Phys. **A698** (2002) 217; R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, JHEP **0109** (2001) 033 [hep-ph/0106347]; Phys. Rev. **C60** (1999) 064902 [hep-ph/9907267]; Nucl. Phys. **B531** (1998) 403 [hep-ph/9804212]; Nucl. Phys. **B484** (1997) 265 [hep-ph/9608322]; Nucl. Phys. **B483** (1997) 291 [hep-ph/9607355].
- [42] B. G. Zakharov, JETP Lett. **63** (1996) 952 [hep-ph/9607440].
- [43] B. G. Zakharov, JETP Lett. **65** (1997) 615 [hep-ph/9704255].
- [44] K. J. Eskola, H. Honkanen, C. A. Salgado and U. A. Wiedemann, Nucl. Phys. **A747** (2005) 511 [hep-ph/0406319]; A. Dainese, C. Loizides and G. Paic, Eur. Phys. J. **C38** (2005) 461 [hep-ph/0406201].
- [45] A. H. Rezaeian and Z. Lu, Nucl. Phys. **A826** (2009) 198 [arXiv:0810.4942]; B. Z. Kopeliovich, A. H. Rezaeian, I. Schmidt, Phys. Rev. **D78** (2008) 114009 [arXiv:0809.4327].
- [46] For a review see: D. d'Enterria, Landolt-Boernstein Vol. 1-23A (Springer Verlag), arXiv:0902.2011.
- [47] F. Dominguez, C. Marquet, A. H. Mueller, B. Wu and B. W. Xiao, Nucl. Phys. **A811** (2008) 197 [arXiv:0803.3234].
- [48] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L. G. Yaffe, JHEP **0607** (2006) 013 [hep-th/0605158].
- [49] S. S. Gubser, Phys. Rev. **D74** (2006) 126005 [hep-th/0605182].
- [50] H. Liu, K. Rajagopal and U. A. Wiedemann, Phys. Rev. Lett. **97** (2006) 182301 [hep-ph/0605178].
- [51] M. Gyulassy, P. Levai and I. Vitev, Phys. Rev. Lett. **85** (2000) 5535 [nucl-th/0005032].
- [52] M. Gyulassy, P. Levai and I. Vitev, Nucl. Phys. **B594** (2001) 371 [nucl-th/0006010].
- [53] M. Gyulassy, I. Vitev, X. N. Wang and B. W. Zhang, Published in Quark Gluon Plasma 3, editors: R. C. Hwa and X. N. Wang, World Scientific, Singapore. In Hwa, R. C. (ed.) *et al.*: Quark gluon plasma, 123-191, nucl-th/0302077.
- [54] C. A. Salgado and U. A. Wiedemann, Phys. Rev. **D68** (2003) 014008 [hep-ph/0302184].
- [55] Y. L. Dokshitzer, V. A. Khoze, and S. I. Troian, J. Phys. **G17** (1991) 1585; V. A. Khoze, W. Ochs, and J. Wosiek, hep-ph/0009298; V. A. Khoze and W. Ochs, Int. J. Mod. Phys. **A12** (1997) 2949, and reference therein.
- [56] D. Kharzeev, E. Levin, M. Nardi and K. Tuchin, Nucl. Phys. **A826** (2009) 230 [arXiv:0809.2933]; Phys. Rev. Lett. **102** (2009) 152301 [arXiv:0808.2954].
- [57] K. Tuchin, Phys. Lett. **B593** (2004) 66 [hep-ph/0401022].
- [58] Y. V. Kovchegov and K. Tuchin, Phys. Rev. **D74** (2006) 054014 [hep-ph/0603055].

- [59] Y. V. Kovchegov, Nucl. Phys. **A692** (2001) 557 [hep-ph/0011252].
- [60] B. Z. Kopeliovich, A. V. Tarasov and A. Schäfer, Phys. Rev. **C59** (1999) 1609 [hep-ph/9808378].
- [61] B. Z. Kopeliovich and A. H. Rezaeian, Int. J. Mod. Phys. E **18** (2009) 1629 [arXiv:0811.2024] and references therein.
- [62] K. J. Golec-Biernat and M. Wusthoff, Phys. Rev. **D60** (1999) 114023 [hep-ph/9903358].
- [63] E. Levin and K. Tuchin, Nucl. Phys. **A693** (2001) 787, [arXiv:hep-ph/0101275]; **A691** (2001) 779 [hep-ph/0012167]; **B573** (2000) 833, [hep-ph/9908317].
- [64] B. Z. Kopeliovich, I. K. Potashnikova, B. Povh and I. Schmidt, Phys. Rev. **D76** (2007) 094020 [arXiv:0708.3636].
- [65] A. Kormilitzin and E. Levin, arXiv:1009.1468.
- [66] E. Gotsman, E. Levin, U. Maor and E. Naftali, Nucl. Phys. **A750** (2005) 391 [hep-ph/0411242].
- [67] J. L. Albacete and C. Marquet, Phys. Lett. **B687** (2010) 174 [arXiv:1001.1378].
- [68] G. Watt and H. Kowalski, Phys. Rev. **D78** (2008) 014016.
- [69] K. Golec-Biernat and A. M. Stasto, Nucl. Phys. **B668** (2003) 345; J. Berger and A. Stasto, arXiv:1010.0671.
- [70] S. Abreu *et al.*, J. Phys. **G35** (2008) 054001 [arXiv:0711.0974].
- [71] V. A. Abramovsky, V. N. Gribov, O. V. Kancheli, Yad. Fiz. **18** (1973) 595-616, , (See in Luca Caneschi, Regge Theory of Low-pT Hadronic Interaction (North-Holland, Amsterdam, 1989)).